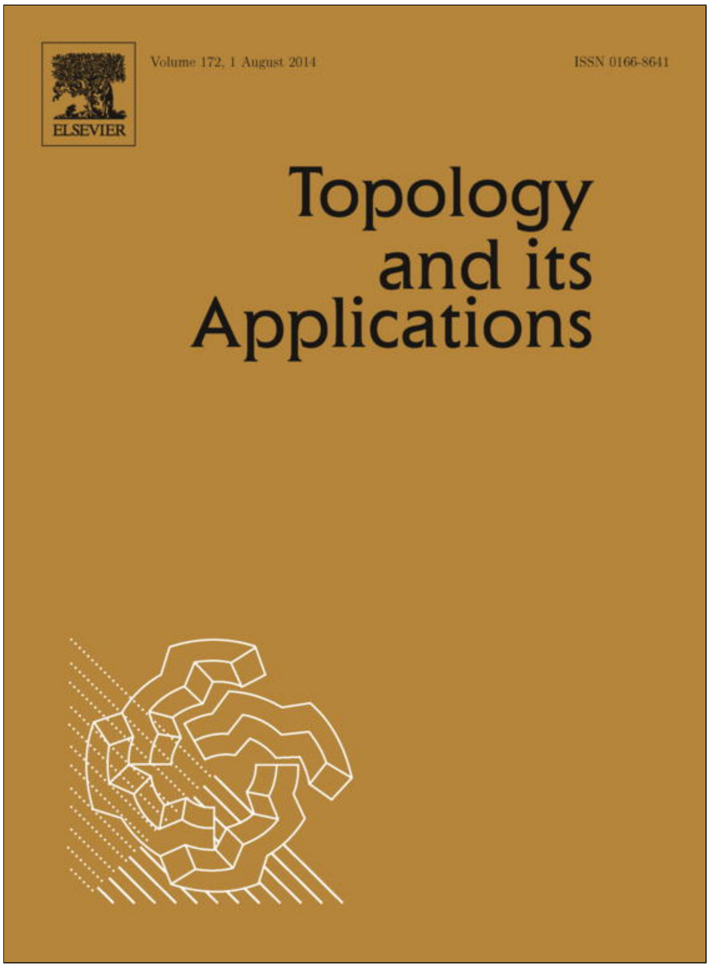
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Submetrizability in semitopological groups ☆

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ABSTRACT

In this paper, we discuss submetrizability in semitopological groups. It gives a positive answer to a question posed in Tkachenko (2013) [15].

1. Introduction

Recall that a *semitopological group* is a group with a topology such that the multiplication in the group is separately continuous. A *paratopological group* is a group with a topology such that the multiplication is jointly continuous. If G is a paratopological group and the inverse operation of G is continuous, then G is called a *topological group*.

A space X is called *submetrizable* if there exists a continuous bijection from X onto a metrizable space. It is known that every Hausdorff topological group G of countable pseudocharacter is submetrizable

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[4, Lemma 6.10.7]. Unlike topological groups, Hausdorff paratopological groups of countable pseudocharacter may fail to be submetrizable. This fact has recently been established by F. Lin and C. Liu in [9, Example 3.3]. However, A.V. Arhangel'skiĭ et al. in [1–3], investigated when a semitopological group (or a paratopological group) is submetrizable (or admits a continuous bijection onto a Hausdorff space with a countable base). Recently, L.-H. Xie and S. Lin [17] discussed submetrizability in paratopological groups.

It is an old problem posed by A.V. Arhangel'skiĭ to find an upper bound for cardinalities of regular Lindelöf spaces of countable pseudocharacter. In topological groups and paratopological groups, A.V. Arhangel'skiĭ's problem has a relatively simple solution. Sanchis and Tkachenko [11, Theorem 2.26] proved that every Hausdorff paratopological group G with $l(G) \cdot \psi(G) \le \omega$ has cardinality less than or equal to 2^ω. In an attempt to supplement the above theorem, the following questions were posed by Tkachenko:

Question 1.1. ([15, Problem 6.17]) Does every Hausdorff (or regular) semitopological group G with $l(G) \cdot \psi(G) \le \omega$ satisfy $|G| \le 2^\omega$?

Question 1.2. ([15, Problem 6.18]) Does a Hausdorff or regular paratopological group G with $l(G) \cdot \psi(G) \le \omega$ admit a continuous bijection onto a Hausdorff space with a countable base?

In this paper, we mainly discuss submetrizability in semitopological groups. Question 1.1 is partially solved and we also give a positive answer to Question 1.2. We establish that: (1) every paracompact semitopological group G with $Hs(G) \cdot \psi(G) \le \omega$ is submetrizable (see Theorem 2.2); (2) every Hausdorff semitopological group G with $Hs(G) \cdot In_r(G) \cdot \psi(G) \le \omega$ admits a continuous bijection onto a Hausdorff space with a countable base, in particular, $|G| \le 2^\omega$; in addition, the diagonal of G is a G_δ-set in $G \times G$ (see Theorem 2.7); (3) every Tychonoff semitopological group G with $Hs(G) \cdot In_r(G) \cdot \psi(G) \le \omega$ admits a weaker separable metrizable topology (see Theorem 2.13). Some results in [1,9,17] are improved.

All spaces in this paper satisfy the T_1 separation axiom. Below $w(X)$, $l(X)$, $\psi(X)$ and $\pi_\chi(X)$ denote the weight, Lindelöf degree, pseudocharacter and π-character of a space X defined, respectively, as follows:

weight: $w(X) = \min\{|\mathcal{U}| : \mathcal{U}$ is a base for $X\} + \omega$;

Lindelöf degree: $l(X) = \min\{\lambda \in \mathrm{Card}$: for every open cover $\mathcal{V}$ of X there is a subfamily $\mathcal{U} \subset \mathcal{V}$ such that $|\mathcal{U}| \le \lambda$ and $\bigcup \mathcal{U} = X\} + \omega$;

pseudocharacter: $\psi(X) = \sup\{\min\{|\mathcal{U}| : \mathcal{U}$ is a family of open subsets of X such that $\bigcap \mathcal{U} = \{x\}\} : x \in X\} + \omega$;

π-character: $\pi_\chi(G) = \sup\{\min\{|\mathcal{B}| : \mathcal{B}$ is a π-base at $x\} : x \in X\} + \omega$.

The reader can consult [4] and [5] for notation and terminology not given here.

2. Main results

Recall that for a Hausdorff semitopological group G with identity e the *Hausdorff number* [14] of G, denoted by $Hs(G)$, is the minimum cardinal number κ such that for every neighborhood U of e in G, there exists a family γ of neighborhoods of e such that $\bigcap_{V \in \gamma} VV^{-1} \subset U$ and $|\gamma| \le \kappa$.

Let X be a space. Then $\Delta = \{(x, x) : x \in X\}$ is a diagonal in $X \times X$. $\Delta(X) = \min\{|\mathcal{U}| : \mathcal{U}$ is a family of open subsets of X^2 such that $\bigcap \mathcal{U} = \Delta\} + \omega$. If $\Delta(X) \le \omega$ for a space X, then X has a G_δ-diagonal.

Proposition 2.1. *$\Delta(G) \le Hs(G) \cdot \psi(G)$ for every Hausdorff semitopological group G.*

Proof. Let $Hs(G) \cdot \psi(G) \le \alpha$. Suppose that e is the neutral element in G and that $\{e\} = \bigcap_{\gamma \in \alpha} U_\gamma$, where U_γ is an open set in G for each $\gamma \in \alpha$. Since $Hs(G) \le \alpha$, there is a family Λ_γ of open neighborhoods of e such that $\bigcap_{V\in\Lambda_\gamma} VV^{-1} \subset U_\gamma$ and $|\Lambda_\gamma| \le \alpha$ for each U_γ. We put $\Lambda = \bigcup_{\gamma\in\alpha} \Lambda_\gamma$. Clearly, we have $|\Lambda| \le \alpha$. One can easily obtain $\{e\} = \bigcap_{V\in\Lambda} VV^{-1}$. We put $U_V = \bigcup_{x\in G} Vx \times Vx$. Then $\Delta = \bigcap_{V\in\Lambda} U_V$, where Δ is the diagonal of $G\times G$. Indeed, assume the contrary. Then there exist distinct a, b in G such that $(a, b) \in U_V$ for each $V \in \Lambda$. We put $y = ab^{-1}$. Then $y \ne e$ and, for $V \in \Lambda$, there exists $x \in G$ such that $(a, b) \in Vx \times Vx$. It follows that $y = ab^{-1} \in Vx(Vx)^{-1} = VV^{-1}$. Hence $y \in \bigcap_{V\in\Lambda} VV^{-1} = \{e\}$, which contradicts $y \ne e$. □

It is known that every Hausdorff paracompact space with a G_δ-diagonal is submetrizable (see [6, Corollary 2.9]). By Proposition 2.1, we have the following theorem.

Theorem 2.2. *Every Hausdorff paracompact semitopological group G with $Hs(G)\cdot\psi(G) \le \omega$ is submetrizable.*

A subset U of a space X is called *regular open* if $U = \mathrm{Int}(\overline{U})$. Similarly, a subset F of a space X is called *regular closed* if $F = \overline{\mathrm{Int}(F)}$. Given a space (X, τ), denote by τ' the topology on X whose base consists of regular open subsets of (X, τ). The space (X, τ') is said to be the *semiregularization* of (X, τ) and is denoted by X_{sr}. It is easy to see that $\tau' \subset \tau$ and that the spaces (X, τ) and (X, τ') have the same regular open and regular closed subsets. The operation of semiregularization was defined by M. Stone in [13] and studied by M. Katetov [8].

Now we give a positive answer to Question 1.2 and show even more:

Corollary 2.3. *Every Hausdorff paratopological group G with $l(G)\cdot\psi(G) \le \omega$ admits a weaker separable metrizable topology.*

Proof. Let G_{sr} be the semiregularization of G. Since G is Hausdorff, it follows from [10, Example 1.9] that G_{sr} is a regular paratopological group. Hence, it is enough to show that G_{sr} admits a weaker separable metrizable topology. One can easily verify that $l(G_{sr})\cdot\psi(G_{sr}) \le \omega$. Hence, from [14, Proposition 2.4] it follows that $Hs(G_{sr})\cdot\psi(G_{sr}) \le l(G_{sr})\cdot\psi(G_{sr}) \le \omega$. Clearly, G_{sr} is paracompact, since G_{sr} is a Lindelöf space. By Theorem 2.2, G_{sr} is submetrizable. Let G'_{sr} be G_{sr} with a weaker metrizable topology. Then the identity map $i : G_{sr} \to G'_{sr}$ is continuous. Hence, G'_{sr} is Lindelöf as a continuous image of G_{sr}. Since G'_{sr} is metrizable, G'_{sr} is a separable metrizable space. □

The second result in Proposition 2.4 improves [14, Proposition 2.2].

Proposition 2.4. *Let G be a Hausdorff semitopological group. Then*

(1) $\psi(G) \le \pi_\chi(G)$;
(2) $Hs(G) \le \pi_\chi(G)$.

Proof. Let γ be a π-base at the neutral element e in G.

(1) We shall show that $\{e\} = \bigcap_{V\in\gamma} VV^{-1}$, which implies that $\psi(G) \le \pi_\chi(G)$. Indeed, for any $y \ne e$, there are two disjoint open sets W_1 and W_2 in G such that $e \in W_1$ and $y \in W_2$. Since G is a semitopological group, there is an open set U containing e such that $U \subset W_1$ and $yU \subset W_2$. From the fact that γ is a π-base at e it follows that there is an open set $V \in \gamma$ such that $V \subset U$. One can easily verify that $y \notin VV^{-1}$ and $e \in VV^{-1}$. Indeed, since $yU \cap U = \emptyset$, we have $y \notin UU^{-1} \supset VV^{-1}$. This implies that $\psi(G) \le |\gamma|$, as required.

(2) For each $V \in \gamma$, fix a point $b_V \in V$. Clearly, Vb_V^{-1} is an open set containing e. For any open set U containing e, we shall show that $\bigcap_{V\in\gamma}(Vb_V^{-1})(Vb_V^{-1})^{-1} \subset U$, which implies that $Hs(G) \le |\gamma|$. Indeed, $\bigcap_{V\in\gamma}(Vb_V^{-1})(Vb_V^{-1})^{-1} = \bigcap_{V\in\gamma} Vb_V^{-1}b_V V^{-1} = \bigcap_{V\in\gamma} VV^{-1} = \{e\} \subset U$, as required. □

Corollary 2.5. *([3, Theorem 2.6]) Every paracompact semitopological group G of countable π-character is submetrizable.*

Proof. The statement directly follows from Theorem 2.2 and Proposition 2.4. $\square$

According to Propositions 2.1 and 2.4, we have the following:

Corollary 2.6. *$\Delta(G) \le \pi_\chi(G)$ for every Hausdorff semitopological group G.*

Let τ be an infinite cardinal and G a semitopological group. Then G is called *left* (resp. *right*) *τ-narrow* [11] if, for every neighborhood U of the identity in G, there exists a subset $K \subset G$ with $|K| \le \tau$ such that $KU = G$ (resp. $KU = G$). The *left index of narrowness $In_l(G)$* and the *right index of narrowness $In_r(G)$* of G are defined as the minimal cardinals $\tau \ge \omega$ such that G is left τ-narrow and right τ-narrow, respectively.

Theorem 2.7. *Every Hausdorff semitopological group G with $Hs(G) \cdot In_r(G) \cdot \psi(G) \le \omega$ admits a continuous bijection onto a Hausdorff space with a countable base, in particular, $|G| \le 2^\omega$. In addition, the diagonal of G is a G_δ-set in $G \times G$.*

Proof. Suppose that e is the neutral element in G and that $\{e\} = \bigcap_{n\in\omega} U_n$, where U_n is an open set in G for each $n \in \omega$. Since $Hs(G) \le \omega$, there is a countable family γ_n of open neighborhoods of e such that $\bigcap_{V\in\gamma_n} VV^{-1} \subset U_n$ for each U_n. We put $\gamma = \bigcup_{n\in\omega} \gamma_n$. Then, obviously, $\{e\} = \bigcap_{V\in\gamma} VV^{-1}$. Since $In_r(G) \le \omega$, for each $V \in \gamma$ one can find a countable subset H_V in G such that $VH_V = G$. We put $M_V = H_V \cup H_V^{-1}$, $M = \bigcup_{V\in\gamma} M_V$ and $H = \bigcup_{n\in\mathbb{N}} M^n$. Then H is a countable subgroup in G.

Claim 1. *$b^{-1}V \cap H \ne \emptyset$ for each $b \in G$ and $V \in \gamma$.*

Since $b^{-1}VH = b^{-1}G = G$, we have $b^{-1}VH \cap H \ne \emptyset$. Thus, there exist $h_1, h_2 \in H$ and $y \in V$ such that $b^{-1}yh_1 = h_2$. That is, $b^{-1}y = h_2h_1^{-1} \in H$. This completes the proof of Claim 1.

We put $\zeta = \{Vh^{-1} : h \in H, V \in \gamma\} \cup \{G \setminus \overline{V}h^{-1} : h \in H, V \in \gamma\}$.

Claim 2. *For any two distinct points a, b in G, there are disjoint $W_1 \in \zeta$ and $W_2 \in \zeta$ such that $a \in W_1$ and $b \in W_2$.*

Since $ab^{-1} \ne e$, there is $V \in \gamma$ such that $ab^{-1} \notin VV^{-1}$. Thus one can easily obtain $ab^{-1}V \cap \overline{V} = \emptyset$.

By Claim 1, one can choose $d \in b^{-1}V \cap H$. Then $ad \notin \overline{V}$. Hence, $a \notin \overline{V}d^{-1}$, that is, $a \in G \setminus \overline{V}d^{-1}$. We put $W_1 = G \setminus \overline{V}d^{-1}$.

From $d \in b^{-1}V$ it follows that $bd \in V$, that is, $b \in Vd^{-1}$. We put $W_2 = Vd^{-1}$. Clearly, W_1 and W_2 are both in ζ, $W_1 \cap W_2 = \emptyset$ and $a \in W_1$, $b \in W_2$. This proves Claim 2.

Now we can use the countable family ζ as a subbase for a new topology τ on G; this topology is, clearly, Hausdorff, and is contained in the original topology of G. The identity map $i : G \to G_\tau$ is a continuous bijection, where G_τ is G with the topology τ.

Since $|X| \le 2^{w(X)}$ for each T_0 space X (see [7, Theorem 3.1]), $|G| = |G_\tau| \le 2^\omega$.

From Proposition 2.1 it follows that the diagonal of G is a G_δ-set in $G \times G$. $\square$

A space X is said to have a *regular G_δ-diagonal* if the diagonal $\Delta = \{(x, x) : x \in X\}$ can be represented as the intersection of the closures of a countable family of open neighborhoods of Δ in $X \times X$. Obviously, any submetrizable space has a regular G_δ-diagonal. Every space with a regular G_δ-diagonal is Hausdorff. Indeed, according to Zenor [18], a space X has a regular G_δ-diagonal if and only if there exists a sequence $\{\mathcal{V}_n : n \in \omega\}$ of open covers of X with the following property:

$(*)$ For any two distinct points x and y in X, there are open neighborhoods O_x and O_y of x and y, respectively, and $k \in \omega$ such that no element of $\mathcal{V}$ intersects both O_x and O_y.

Remark 2.8. (1) One can replace the condition '$Hs(G) \cdot In_r(G) \cdot \psi(G) \leq \omega$' by '$Hs(G) \cdot In_l(G) \cdot \psi(G) \leq \omega$' in Theorem 2.7;

(2) According to the proof of Theorem 2.7, one can easily obtain that every Hausdorff semitopological group G admits a continuous bijection onto a Hausdorff space X with $w(X) \leq Hs(G) \cdot In_r(G) \cdot \psi(G)$, in particular, $|G| \leq 2^{Hs(G) \cdot In_r(G) \cdot \psi(G)}$. It is worth mentioning that I. Sánchez [12] independently proved that $|G| \leq 2^{Hs(G) \cdot In_r(G) \cdot \psi(G)}$ for every Hausdorff semitopological group G.

(3) In view of Theorem 2.7, it is natural to ask whether every Hausdorff space with a countable base has a G_δ-diagonal. We do not known the answer to this question. However, there exists a nonsubmetrizable and second-countable Hausdorff space with a G_δ-diagonal.

Let $X = \mathbb{R}^+ \cup \{a\} \cup \{b\}$, where $\mathbb{R}^+ = [0, +\infty)$ and a, b are two distinct points not in $\mathbb{R}^+$. We topologize X as follows: $\mathbb{R}^+$ has the usual topology and is an open subspace of X; a basic neighborhood of $a \in X$ has the form

$$O_j(a) = \{a\} \cup \bigcup_{i=j}^{\infty} (2i, 2i+1), \quad \text{where } j \in \omega;$$

a basic neighborhood of $b \in X$ has the form

$$O_k(b) = \{b\} \cup \bigcup_{i=k}^{\infty} (2i-1, 2i), \quad \text{where } k \in \omega.$$

Viglino [16] showed that the space X is Hausdorff. It is clear that X is second-countable. We put $\mathcal{U}_n = \{O_n(a)\} \cup \{O_n(b)\} \cup \{B_{1/n}(x) : x \in \mathbb{R}^+\}$, where $B_{1/n}(x) = \{y : |y - x| < 1/n\}$. Then $\{\mathcal{U}_n : n \in \omega\}$ is a G_δ-diagonal sequence for X. We claim that X does not have a regular G_δ-diagonal, which shows that X is not submetrizable. Indeed, let $\{\mathcal{V}_n : n \in \omega\}$ be a sequence of open covers of X. We take two neighborhoods $O_j(a)$ and $O_k(b)$ of a and b, respectively. Then there exists an integer $m > \max\{j, k\}$. For every $n \in \omega$ and $U_n(m) \in \mathcal{V}_n$ with $k \in U_n(m)$, $U_n(m)$ intersects both $O_j(a)$ and $O_k(b)$, which shows that X does not have a regular G_δ-diagonal.

The following result gives a partial answer to Question 1.1 and even more:

Corollary 2.9. *Every Hausdorff (resp. regular) semitopological group G with $Hs(G) \cdot l(G) \cdot \psi(G) \leq \omega$ admits a continuous bijection onto a Hausdorff (resp. regular) space with a countable base, in particular, $|G| \leq 2^\omega$.*

Proof. By Theorem 2.7, clearly, we have $|G| \leq 2^\omega$.

When the semitopological group G satisfies the Hausdorff separation axiom, the statement directly follows from Theorem 2.7.

Suppose that G is regular. From Theorem 2.7 it follows that G has a G_δ-diagonal. Hence, from the fact that every paracompact space with a G_δ-diagonal is submetrizable [6, Corollary 2.9] it follows that G is a submetrizable space, since every regular Lindelöf space is paracompact. Suppose that G' is G with a weaker metrizable topology. Then the identity $i : G \to G'$ is continuous. Since every continuous image of a Lindelöf space is Lindelöf, G' is a Lindelöf metrizable space. Hence, G' is a metrizable space with a countable base. This completes the proof. $\square$

The following result gives another partial answer to Question 1.1.

Corollary 2.10. *Every Hausdorff (resp. regular) semitopological group G with $l(G) \cdot \pi_\chi(G) \leq \omega$ admits a continuous bijection onto a Hausdorff (resp. regular) space with a countable base, in particular, $|G| \leq 2^\omega$.*

Proof. This follows from Proposition 2.4 and Corollary 2.9. □

Corollary 2.11. *([11, Theorem 2.26]) Every Hausdorff paratopological group G with $l(G) \cdot \psi(G) \leq \omega$ has cardinality less than or equal to 2^ω. In addition, the diagonal of G is a G_δ-set in $G \times G$.*

Proof. It is well known that $Hs(G) \leq l(G)$ for every Hausdorff paratopological group G [14, Proposition 2.4]. Clearly, $In_r(G) \leq l(G)$. Hence, the statement directly follows from Theorem 2.7. □

F. Lin and C. Liu [9, Theorem 3.6] proved that every regular ω-narrow first-countable paratopological group G admits a continuous bijection onto a Hausdorff space with a countable base. Recently, L.-H. Xie and S. Lin [17, Proposition 2.2] weakened 'regular' to 'Hausdorff'. The following result generalizes this result to the class of semitopological group.

Corollary 2.12. *Every Hausdorff left (right) ω-narrow semitopological group G with countable π-character admits a continuous bijection onto a Hausdorff space with a countable base.*

Proof. This follows from Proposition 2.4 and Theorem 2.7. □

A result similar to Theorem 2.7 holds in the class of Tychonoff spaces.

Theorem 2.13. *Every Tychonoff semitopological group G with $Hs(G) \cdot In_r(G) \cdot \psi(G) \leq \omega$ admits a weaker separable metrizable topology.*

Proof. The proof is very close to the proof of Theorem 2.7. Since the space G is Tychonoff, all elements of γ in the proof of Theorem 2.7 can be chosen to be cozero-sets. Since translations are homeomorphisms, all elements of the family $\zeta_1 = \{Vh^{-1} : h \in H, V \in \gamma\}$ are also cozero-sets. Therefore, for every $W \in \zeta_1$, we can fix a continuous function $f_W : G \to \mathbb{R}$ such that $W = f_W^{-1}(\mathbb{R} \setminus \{0\})$. Then $\mathscr{F} = \{f_W : W \in \zeta_1\}$ is a countable family of continuous functions on G separating points of G (see the proof of Theorem 2.7). Hence, the diagonal product of functions in $\mathscr{F}$ is a one-to-one continuous mapping of G onto a separable metrizable space. □

Clearly, one can replace the condition '$Hs(G) \cdot In_r(G) \cdot \psi(G) \leq \omega$' by '$Hs(G) \cdot In_l(G) \cdot \psi(G) \leq \omega$' in Theorem 2.13. Hence, according to Proposition 2.4 and Theorem 2.13, we have the following:

Corollary 2.14. *([1, Theorem 2.4]) Every Tychonoff left ω-narrow semitopological group of countable π-character admits a weaker separable metrizable topology.*

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