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A Mapping Theorem On *sn*-metrizable Spaces

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Abstract: A space is called an *sn*-metrizable space if it is a regular space with a σ-locally finite *sn*-network. In this paper an expandable property of k-semistratifiable spaces is discussed, it is shown that *sn*-metrizability is preserved by closed sequence-covering mappings, and some related examples of mapping properties on *sn*-metrizable spaces are given.

Key words: k-semistratifiable spaces; *sn*-metrizable spaces; α_4-spaces; sequence-covering mappings; closed mappings

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0 Introduction

In this paper all spaces are regular and T_1, all mappings are continuous and onto. Every metric space is a g-metrizable space, and every g-metrizable space is an *sn*-metrizale space. *sn*-metrizable spaces inherit some mapping properties from metric spaces or g-metrizable spaces[3]. It is well-known that metrizability is preserved by open and closed mappings. Every open mapping of metric spaces is sequence-covering[8]. After Yan Pengfei, Lin Shou and Jiang Shouli[10] proved metrizability is preserved by closed sequence-covering mappings, Liu Chuan[5] showed that g-metrizability is also preserved by closed sequence-covering mappings, which gives an affirmative answer to the question 3.4.5 in [4]. In this paper it is shown that *sn*-metrizability is preserved by closed sequence-covering mappings, which improves some related mapping theorems.

1 Some Lemmas

First, we discuss some generalized metric properties with respect to *sn*-metrizable spaces. Recalled some related concepts. Refer to [4] for terms which are not defined here.

Definition 1.1[7] A space X is said to be a *k-semistratifiable space* if for each open subset U of X there is a sequence $\{F(n,U)\}_{n\in\mathbb{N}}$ of closed subsets of X such that

(1) $U=\bigcup_{n\in\mathbb{N}}F(n,U)$;

(2) If $V\subset U$, then $F(n,V)\subset F(n,U)$;

(3) If a compact subset $K\subset U$, then $K\subset F(m,U)$ for some $m\in\mathbb{N}$.

The correspondence $U\to\{F(n,U)\}_{n\in\mathbb{N}}$ is said to be a *k-semistratification* for the space X.

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Let X be a space. For $P \subset X$, P is a *sequential neighborhood* of x in X if every sequence converging to x is eventually in P. P is a sequentially open subset of X if P is a sequential neighborhood of x in X for each $x \in P$. P is a sequentially closed subset of X if $X \setminus P$ is sequentially open. X is said to be a *sequential space*[2] if each sequentially open subset is open in X.

Let $\mathcal{P}$ be a family of subsets of a space X. $\mathcal{P}$ is *discrete* in X if there is a neighborhood U of x in X such that U meets at most some element of $\mathcal{P}$ for each $x \in X$. $\mathcal{P}$ is *closure-preserving* in X if $\overline{\cup \mathcal{P}'} = \cup\{\overline{P} : P \in \mathcal{P}'\}$ for each $\mathcal{P}' \subset \mathcal{P}$. $\mathcal{P}$ is *s-closure-preserving* in X if $\cup \mathcal{P}'$ is sequential closed in X for each $\mathcal{P}' \subset \mathcal{P}$. $\mathcal{P}$ is *s-discrete* in X if $\mathcal{P}$ is disjoint and s-closure-preserving in X. A subset D of X is *s-discrete* if $\{\{x\} : x \in D\}$ is s-discrete in X. Obviously, a discrete(resp. closure-preserving) family of closed subsets of X is s-discrete(resp. s-closure-preserving).

Lemma 1.2 Let X be a k-semistratifiable space. Then for each subset W of X there is a sequence $\{H(n, W)\}_{n \in \mathbb{N}}$ of closed subsets of X such that

(1) $H(n, W) \subset H(n+1, W) \subset W$;

(2) If $V \subset W$, then $H(n, V) \subset H(n, W)$;

(3) If W is a sequential neighborhood of x, then every sequence converging to x is eventually in $H(m, W)$ for some $m \in \mathbb{N}$;

(4) If $\{G_\alpha : \alpha \in \Lambda\}$ is a disjoint family of subsets of X and $n \in \mathbb{N}$, then $\{H(n, G_\alpha) : \alpha \in \Lambda\}$ is a discrete family in X.

Proof Let $U \to \{F(n, U)\}_{n \in \mathbb{N}}$ be a k-semistratification for X. We can assume that each $F(n, U) \subset F(n+1, U)$. For each $n \in \mathbb{N}, x \in X$, define that $g(n, x) = X \setminus F(n, X \setminus \{x\})$, then $g(n, x)$ is open in X and $x \in g(n+1, x) \subset g(n, x)$. For each $n \in \mathbb{N}, W \subset X$, put $H(n, W) = X \setminus \bigcup_{x \in X \setminus W} g(n, x)$, then $H(n, W)$ is closed in X and satisfies the conditions (1) and (2).

Let W be a sequential neighborhood of x in X and a sequence $\{x_n\}$ converges to x. If (3) is not hold, then for each $i \in \mathbb{N}$, there is $x_{n_i} \in X \setminus H(i, W)$, thus there is $y_i \in X \setminus W$ such that $x_{n_i} \in g(i, y_i)$. Let U be an open neighborhood of x. There are $k, m \in \mathbb{N}$ such that $\{x_{n_i} : i \geq k\} \subset F(m, U)$, thus $y_i \in U$ for each $i \geq \max\{k, m\}$, hence the sequence $\{y_i\}$ converges to x, a contradiction because W is a sequential neighborhood of x.

Let $\{G_\alpha : \alpha \in \Lambda\}$ be a disjoint family of subsets of X and $n \in \mathbb{N}$. For each $x \in X$, take $V = X \setminus H(n, \cup\{G_\alpha : \alpha \in \Lambda \text{ and } x \notin G_\alpha\})$, then V is an open neighborhood of x in X and $V \cap H(n, G_\alpha) = \emptyset$ if $x \notin G_\alpha$. Hence $\{H(n, G_\alpha) : \alpha \in \Lambda\}$ is a discrete family of subsets of X.

Lemma 1.3 Let X be a k-semistratifiable space. Then each s-discrete subset of X has an s-discrete extension of sequential neighborhoods in X.

Proof Let X be a k-semistratifiable space, and $W \to \{H(n, W)\}_{n \in \mathbb{N}}$ a correspondence of X satisfying all conditions in Lemma 1.2.

Let $\{x_\alpha : \alpha \in \Lambda\}$ be an s-discrete subset of X. We shall prove that there is an s-discrete family $\{W_\alpha : \alpha \in \Lambda\}$ such that each W_α is a sequential neighborhood of x_α in X. For each $\alpha \in \Lambda$, let $L_\alpha = \{x_\beta : \beta \in \Lambda \setminus \{\alpha\}\}$, $G_\alpha = \bigcup_{n \in \mathbb{N}}(H(n, X \setminus L_\alpha) \setminus H(n, X \setminus \{x_\alpha\}))$. Then $\{G_\alpha : \alpha \in \Lambda\}$ is a disjoint family of subsets of X. We shall first prove that G_α is a sequential neighborhood of x_α for each $\alpha \in \Lambda$.

Let S be a sequence converging to some x_α in X. Since L_α is sequential closed and $x_\alpha \notin L_\alpha$, S is eventually in $H(m, X \setminus L_\alpha)$ for some $m \in \mathbb{N}$ by Lemma 1.2, and $x_\alpha \notin H(m, X \setminus \{x_\alpha\})$, so we can assume that S is eventually in $H(m, X \setminus L_\alpha) \setminus H(m, X \setminus \{x_\alpha\}) \subset G_\alpha$, hence G_α is a sequential neighborhood of x_α.

For each $n \in \mathbb{N}, \alpha \in \Lambda$, $x_\alpha \notin H(n, X \setminus \{x_\alpha\})$. By the regularity, there is an open subset $V_\alpha(n)$ such that $x_\alpha \in V_\alpha(n) \subset \overline{V_\alpha(n)} \subset X \setminus H(n, X \setminus \{x_\alpha\})$. Put $F_\alpha(n) = H(n, G_\alpha) \cap \overline{V_\alpha(n)}$, $W_\alpha = \bigcup_{n\in\mathbb{N}} F_\alpha(n)$. Then W_α is a sequential neighborhood of x_α. In fact, if S is a sequence converging to x_α in X, S is eventually in $H(m, G_\alpha)$ for some $m \in \mathbb{N}$ by Lemma 1.2, and S is eventually in $V_\alpha(m)$, thus S is eventually in $F_\alpha(m) \subset W_\alpha$.

Let $\mathcal{W} = \{W_\alpha : \alpha \in \Lambda\}$. Then $\mathcal{W}$ is a disjoint family because of each $W_\alpha \subset G_\alpha$. To complete the proof of the Lemma, it suffices to show that $\mathcal{W}$ is an s-closure-preserving family, i. e., $\bigcup_{\alpha\in\Lambda'} W_\alpha$ is sequential closed in X for each $\Lambda' \subset \Lambda$. Let S be a sequence converging to $x \notin \bigcup_{\alpha\in\Lambda'} W_\alpha$. Then $x \notin \{x_\alpha : \alpha \in \Lambda'\}$, S is eventually in $H(m, X \setminus \{x_\alpha : \alpha \in \Lambda'\}$ for some $m \in \mathbb{N}$, and $H(m, X \setminus \{x_\alpha : \alpha \in \Lambda'\}) \cap F_\alpha(n) \subset H(m, X \setminus \{x_\alpha\}) \cap \overline{V_\alpha(n)} = \emptyset$ for each $\alpha \in \Lambda'$ and $n \geq m$. By Lemma 1.2, $\{H(n, G_\alpha) : \alpha \in \Lambda\}$ is a discrete family in X for each $n \in \mathbb{N}$, so $\{F_\alpha(n) : \alpha \in \Lambda\}$ is a discrete family of closed subsets of X. Put $E(m, \Lambda') = \bigcup_{\alpha\in\Lambda', n<m} F_\alpha(n)$. Then $E(m, \Lambda')$ is closed and $x \notin E(m, \Lambda')$, thus S is eventually in $X \setminus E(m, \Lambda')$. Hence S is eventually in $X \setminus \bigcup_{\alpha\in\Lambda'} W_\alpha$, and $\bigcup_{\alpha\in\Lambda'} W_\alpha$ is sequential closed in X.

Remark 1.4 A locally compact *Moore space* can not be of the expandable property in Lemma 1.3. For example, the well-known Gillman-Jerison space $\psi(\mathbb{N}) = \mathbb{N} \cup \mathcal{A}$ (see [4]), where $\mathcal{A}$ is an almost disjoint and maximal family of $\mathbb{N}$. The $\mathcal{A}$ is a discrete closed subspace, it has not any s-discrete extension of sequential neighborhoods in $\psi(\mathbb{N})$.

Lemma 1.5[4] Let $f : X \to Y$ be a closed mapping. Let K be a countably compact subset of Y, and let $S = \{x_n : n \in \mathbb{N}\}$ be a sequence in $f^{-1}(K)$ such that $f(x_m) \neq f(x_n)$ if $m \neq n$. If each point of X is a G_δ-set, then there exists a convergent subsequence of S.

Let $f : X \to Y$ be a mapping. f is a sequence-covering mapping[8] if L is a convergent sequence in Y, there is a convergent sequence M in X such that $f(M) = L$. A perfect mapping of a metric space may not be sequence-covering. For example, let $X = (\{0\} \cup \{\frac{1}{2n} : n \in \mathbb{N}\}) \oplus (\{0\} \cup \{\frac{1}{2n} - 1 : n \in \mathbb{N}\})$, $Y = \{0\} \cup \{\frac{1}{n} : n \in \mathbb{N}\}$. X, Y are endowed with the subspace topology of real line $\mathbb{R}$, and let $f : X \to Y$ be the obvious mapping. Then f is a non-sequence-covering, perfect mapping.

2 sn-metrizable Spaces

Call a subspace of a space X a *fan*(at a point $x \in X$) if it consists of a point x, and a countably infinite family of disjoint sequences converging to x. Call a subset of a fan a *diagonal* if it is a sequence meeting infinitely many of the sequence converging to x and converges to some point in the fan. A space X is an α_4-*space* if every fan at x of X has a diagonal converging to x (see [4]).

Theorem 2.1 Let $f : X \to Y$ be a closed sequence-covering mapping, and X a k-

semistratifiable space. If X is an α_4-space, so does Y.

Proof If Y is not an α_4-space, there is a fan $\{y\}\cup\{y_i(n): i,n\in\mathbb{N}\}$ at some y in Y without a diagonal converging to y, where each $y_i(n)\to y$ as $i\to\infty$. For each $k\in\mathbb{N}$, let $L_k=\{y_i(n): i\in\mathbb{N}, n\le k\}$. Then L_k is a sequence converging to y. Let M_k be a sequence of X converging to $u_k\in f^{-1}(y)$ with $f(M_k)=L_k$, we rewrite $M_k=\{x_i(n,k): i\in\mathbb{N}, n\le k\}$ with each $f(x_i(n,k))=y_i(n)$.

Case 1 $\{u_k: k\in\mathbb{N}\}$ is finite.

There are a $k_0\in\mathbb{N}$ and an infinite subset $\mathbb{N}'$ of $\mathbb{N}$ such that $M_k\to u_{k_0}$ for each $k\in\mathbb{N}'$, then $\{u_{k_0}\}\cup\{x_i(k,k): i\in\mathbb{N}, k\in\mathbb{N}'\}$ is a fan at u_{k_0} in X. Thus it has a diagonal converging to u_{k_0} because X is an α_4-space, so the fan $\{y\}\cup\{y_i(n): i,n\in\mathbb{N}\}$ has a diagonal converging to y, a contradiction.

Case 2 $\{u_k: k\in\mathbb{N}\}$ has a non-trivial convergent sequence in X.

Without loss of generality, we assume that $u_k\to u\in f^{-1}(y)$ as $k\to\infty$. Let $\{U_m\}$ be a sequence of open subsets of X with $\overline{U_{m+1}}\subset U_m$, and $\{u\}=\bigcap_{m\in\mathbb{N}}U_m$. Fix $n,m\in\mathbb{N}$, there is a $k_m\ge n$ such that $u_{k_m}\in U_m$ because the sequence $\{u_k\}$ converges to u, there is an $i_m\in\mathbb{N}$ such that $x_{i_m}(n,k_m)\in U_m$ because the sequence $\{x_i(n,k_m)\}_i$ converges to u_{k_m}. We can assume that each $i_m<i_{m+1}$. Then $f(x_{i_m}(n,k_m))=y_{i_m}(n)$. Since f is closed, any subsequence of the sequence $\{x_{i_m}(n,k_m)\}_m$ has a convergent subsequence in X by Lemma 1.5, and u is the unique accumulation of the sequence $\{x_{i_m}(n,k_m)\}_m$, thus $x_{i_m}(n,k_m)\to u$ as $m\to\infty$. Hence $\{u\}\cup\{x_{i_m}(n,k_m): n,m\in\mathbb{N}\}$ is a fan at u in X, so it has a diagonal converging to u, a contradiction.

Case 3 $\{u_k: k\in\mathbb{N}\}$ has not any non-trivial convergent sequence in X.

Then $\{u_k: k\in\mathbb{N}\}$ is s-discrete in X. By Lemma 1.3, there is an s-discrete family $\{W_k: k\in\mathbb{N}\}$ such that each W_k is a sequential neighborhood of u_k. Since $\{x_i(1,k)\}_i$ converges u_k, there is an $i_k\in\mathbb{N}$ such that $x_{i_k}(1,k)\in W_k$. We can assume that each $i_k<i_{k+1}$, then $\{f(x_{i_k}(1,k))\}$ is a subsequence of $\{y_i(1)\}$, thus $\{x_{i_k}(1,k)\}$ has a convergent subsequence by Lemma 1.5, a contradiction.

Definition 2.2 Let $\mathcal{P}=\bigcup_{x\in X}\mathcal{P}_x$ be a cover of a space X such that for each $x\in X$,

(1) $\mathcal{P}_x$ is a network of x in X;

(2) If $U,V\in\mathcal{P}_x$, then $W\subset U\cap V$ for some $W\in\mathcal{P}_x$.

$\mathcal{P}$ is called a *weak base*[1] for X if whenever $G\subset X$ satisfying for each $x\in G$ there is $P\in\mathcal{P}_x$ with $P\subset G$, then G is open in X; $\mathcal{P}$ is called an *sn-network*[4] for X if each element of $\mathcal{P}_x$ is a sequential neighborhood of x in X for each $x\in X$. A space X is called a *g-metrizable space*[9] (resp. an *sn-metrizable space*[3]) if it has a σ-locally finite weak base(resp. sn-network).

Let $\mathcal{P}$ be a family of subsets of a space X. $\mathcal{P}$ is called a *cs-network*[9] for X if whenever a sequence $\{x_n\}$ converges to $x\in U$ with U open in X there exists a $P\in\mathcal{P}$ such that $\{x_n\}$ is eventually in P and $P\subset U$. A space X is called an *ℵ-space* if it has a σ-locally finite cs-network.

Remark 2.3 For a space X, bases ⇒ weak bases ⇒ sn-networks ⇒ cs-networks[4]. It is known that

(1) Metric spaces ⇔ g-metrizable spaces + Fréchet spaces[9];

(2) g-metrizable spaces $\Leftrightarrow$ sn-metrizable spaces + sequential spaces[3];

(3) sn-metrizable spaces $\Leftrightarrow$ $\aleph$-spaces + α_4-spaces[4];

(4) $\aleph$-spaces $\Leftrightarrow$ spaces with a σ-hereditarily closure-preserving cs-network[11];

(5) $\aleph$-spaces $\Rightarrow$ k-semistratifiable spaces[7].

Theorem 2.4 sn-metrizability is preserved by closed sequence-covering mappings.

Proof Let $f : X \to Y$ be a closed sequence-covering mapping, here X is an sn-metrizable space. Let $\mathcal{B}$ be a σ-locally finite sn-network for X. Put $\mathcal{P} = \{f(B) : B \in \mathcal{B}\}$. Then $\mathcal{P}$ is a σ-hereditarily closure-preserving cs-network for Y because f is a closed sequence-covering mapping. Thus Y is an $\aleph$-space. By Theorem 2.1, Y is an α_4-space. Thus Y is an sn-metrizable space.

Rremark 2.5 Metric spaces or $\aleph$-spaces are not preserved by closed mappings[4], and g-metrizable spaces or sn-metrizable spaces are not preserved by perfect mappings[3]. $\aleph$-spaces are preserved by closed sequence-covering mappings by the proof of Theorem 2.4.

Corollary 2.6[4, 5] Metrizability or g-metrizability is preserved by closed sequence-covering mappings.

Proof Let $f : X \to Y$ be a closed sequence-covering mapping. Suppose that X is a metrizable space(resp. g-metrizable space). Then Y is an sn-metrizable space by Theorem 2.4. And Y is a Fréchet space(resp. sequential space) because f is closed, thus Y is a metrizable space(resp. g-metrizable space).

In the final, some related counterexamples of mapping properties on sn-metrizable spaces are given.

Remark 2.7 There is a closed sequence-covering mapping of a metric space which is not open. Let $X = (\{0\} \cup \{\frac{1}{n} : n \in \mathbb{N}\}) \oplus (\{0\} \cup \{\frac{1}{2n} : n \in \mathbb{N}\})$, $Y = \{0\} \cup \{\frac{1}{n} : n \in \mathbb{N}\}$. X, Y are endowed with the subspace topology of $\mathbb{R}$. Let $f : X \to Y$ be the obvious mapping. Then f is a non-open, closed sequence-covering mapping.

Remark 2.8 It has been shown that every closed sequence-covering mapping of metric spaces is almost open[10]. A mapping $f : X \to Y$ is said to be *almost open* if for each $y \in Y$ there is a $x \in X$ such that the image of each neighborhood of x in X under f is a neighborhood of y in Y. There is a perfect sequence-covering mapping of an sn-metrizable space which is not almost open. Let X_1 and X_2 be respectively subspaces $\mathbb{N} \cup \{p_1\}$ and $\mathbb{N} \cup \{p_2\}$ of the Stone-Čech compactification $\beta\mathbb{N}$, here $p_1, p_2 \in \beta\mathbb{N} \setminus \mathbb{N}$. Let $X = X_1 \oplus X_2$. Then each $\{x\}$ is sequentially open in X because X has not any non-trivial convergent sequence, thus $\{\{x\} : x \in X\}$ is a countable sn-network for X, so X is an sn-metrizable space. Put $A = \{p_1, p_2\}$. Let Y be the quotient space X/A, and $q : X \to Y$ the quotient mapping. Then q is a perfect mapping. q is sequence-covering because each convergent sequence in Y is trivial. Since $q(X_1)$ and $q(X_2)$ are not a neighborhood of $q(A)$ in Y, it is easy check that q is not almost-open.

Remark 2.9 It is well-known that suppose X is a metric space and $f : X \to Y$ is a closed mapping, then Y is a metric space if and only if each $\partial f^{-1}(y)$ is compact(Hanai-Morita-Stone Theorem[4]). Metrizability can not be replaced by sn-metrizability in the result. In fact, let S_2 and S_ω denote respectively the *Arens' space* and *sequential fan*[4]. Then S_ω is a perfect image of sn-metrizable space S_2, and S_ω is not an sn-metrizable space. On the other hand, let X be a subspace of the Stone-Čech compactification $\beta\mathbb{N}$ with $\mathbb{N} \subset X$ and $|X \setminus \mathbb{N}| = \aleph_0$. Then a family $\{\{x\} : x \in X\}$ is a countable sn-network for X, thus X is an sn-metrizable space. Define a natural quotient mapping $q : X \to \frac{X}{C}$ with $C = X \setminus \mathbb{N}$. Then q is closed and the quotient space X/C is a metrizable space because it is homeomorphic to the subspace $\{0\} \cup \{\frac{1}{n} : n \in \mathbb{N}\}$ of $\mathbb{R}$. But $\partial q^{-1}([C]) = C$ is not compact in X.

Remark 2.10 Liu Chuan[5] has shown that a space is metrizable if and only if its every perfect image is g-metrizable, which gives an affirmative answer to a question posed by A. Arhangel' skii in Ohio University topology seminar. But the result is not hold if g-metrizability is replaced by sn-metrizability. In fact, let X be the subspaces $\mathbb{N} \cup \{p\}$ of the Stone-Čech compactification $\beta\mathbb{N}$, here $p \in \beta\mathbb{N} \setminus \mathbb{N}$. Then X is an sn-metrizable space and every perfect image of X is sn-metrizable because every compact subset of X is finite and sn-metrizability is preserved by closed finite-to-one mappings[3].

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sn 可度量化空间的映射定理

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摘要：具有 σ 局部有限 sn 网的正则空间称为 sn 可度量化空间. 本文讨论了 k 半层空间的可扩性质，证明了序列覆盖的闭映射保持 sn 可度量化空间，同时给出与 sn 可度量化空间的映射性质相关的几个例子.

关键词：k 半层空间； sn 可度量空间； α_4 空间；序列覆盖映射；闭映射